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Transverse Coupling in Fiber Optics Part IV: Crosstalk

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We evaluate the crosstalk between adjacent cores in an optical fiber that results from electromagnetic coupling. Means of reducing it are discussed. We find that a 0.5-μm-thick layer of silver can, in principle, reduce the crosstalk from −20 to −130 dB without significant increase of the loss. These theoretical results are obtained for two identical single-mode dielectric slabs. In reality, the slabs are not rigorously identical. Longitudinal fluctuations of slab thickness reduce the crosstalk by at least 40 dB. The slab spacing can accordingly be reduced from, typically, 11 to 6 μm for a constant crosstalk. If the slabs are made dissimilar with a relative difference in thickness of 10 percent, the spacing can be reduced further, to approximately 1.5 times the slab thickness. For example, a 15-μm spacing is required between single-mode dissimilar slabs if the nominal slab thickness is 10 μm, provided scattering can be neglected.

I. INTRODUCTION

In multichannel communication systems, crosstalk between channels is a problem that must be considered. Typically, the crosstalk should be less than −20 dB. This means that, if an optical power of 1 mW is fed into one optical guide of a cable, no more than 10 μW should be transferred into the other guides. Let us assume a typical link length of 10 km. The crosstalk measured over a 1-km-long fiber should be less than −40 dB if the power transfer is proportional to the square of the fiber length, less than −30 dB if the power transfer is proportional to the fiber length, and less than −20 dB if the power transfer is independent of the fiber length. As we shall see, the first power law is applicable to identical uniform fibers, the second to nominally identical irregular fibers, and the third to uniform dissimilar fibers.

In optical fibers, the field decays exponentially in the cladding. Therefore, a modest increase in spacing between adjacent fibers is usually sufficient to reduce the optical coupling to tolerable values.

Yet, in some cases, one needs to minimize the cross section of the cable and the spacing between adjacent fibers. Let us briefly discuss a few relevant applications. The need for minimizing the distance between *single-mode* cores in a fiber does not arise in communication systems presently envisioned for the following reasons: The fiber diameter is required to be large (e.g., larger than about 50 μm) so that the fiber is able to sustain mechanical tensions. Thus, quite a few cores can be accommodated within the fiber diameter with sufficient spacing. Furthermore, the capacity of single-mode fibers is so large there is little incentive to introduce more than one core in the same cladding. The problem of coupling between single-mode fibers (or between fibers carrying few modes) does arise, however, when one tries to increase the image-transmission capacity of a fiber bundle up to the diffraction limit, each core carrying one bit of image information. Crosstalk (image blurring) is minimized if adjacent cores are made dissimilar. However, geometrical irregularities may restore a large coupling between closely spaced cores. (This, incidentally, raises the possibility that measurement of the coupling between dissimilar, closely spaced cores gives useful information on the spectral density of the core irregularities.) The problem of coupling between single-mode dielectric waveguides also arises in integrated optics and in biology in the study of the optical behavior of the retina. The results that we present are general. They are therefore applicable, in principle, to multimode, as well as to single-mode, fibers. However, in practical multimode fibers, slow longitudinal variations of the core dimensions make the propagation constants of the modes of one core sweep randomly through the propagation constants of the modes of the other core. Thus, an averaging takes place that cannot be ignored. The problem of coupling between highly multimoded cores will be only briefly discussed.

The shielding method discussed in this paper consists of the introduction of a layer of metal, typically silver, between the adjacent optical waveguides. A reservation is in order: In some communication systems, metallic layers may be undesirable because they detract from the all-dielectric-cable properties. Shielding between adjacent fibers can be provided alternatively by low-refractive-index plastics such as Teflon® FEP ($n \approx 1.32$) that cause the optical field to decay faster than in the cladding material. The reduction in coupling, however, is much smaller than that provided by metals. Plastic materials can be made very lossy by impregnating them with dyes. High losses, however, are much less effective than small refractive indices in reducing evanescent wave coupling. Therefore, we shall consider mainly metallic layers. The practicality of metallic shields remains an open question.

In the first part of this article series,[1] a general and simple expression of the coupling between two lossy open waveguides was derived. Our formulation requires that only the normalized fields of the individual waveguides along a contour be known. In the present paper, we evaluate in detail the crosstalk between two parallel slabs caused by the electromagnetic coupling and means of reducing it. The crosstalk between two optical slabs has been evaluated by Marcuse,[2] although, in Marcuse's work, the slabs are assumed identical. In reality, unavoidable fluctuations in the slab dimensions reduce the crosstalk, as we shall see, by more than 40 dB. Marcuse has also evaluated the reduction of crosstalk provided by a layer of absorbing material located between the slabs. He found that the waveguide loss increases to intolerably high values before any significant reduction in coupling can be obtained. We find that, if the intermediate layer is metallic, the coupling can be drastically reduced without any significant increase of the waveguide loss. This discrepancy results from the fact that, for metallic layers, the permittivity is negative. For very dissimilar media, the first-order perturbation used by Marcuse is not applicable. In the present paper, we assume that the perturbation caused by the intermediate layer on the propagation is small, but we do not assume that the field in that intermediate layer is close to the field that would exist in the absence of the layer.

In Section II, we evaluate the crosstalk between optical waveguides when the axial wave numbers (or propagation constants) of the isolated guides fluctuate along the system axis. In Section III, we evaluate the spacing between slabs corresponding to a given crosstalk. In Section IV, the transmission is evaluated of a metallic layer under evanescent wave excitation and the crosstalk reduction. In Section V, we evaluate the loss that results from the introduction of a metallic layer near a slab waveguide. In Section VI, a simple approximate formula is given for the coupling between oversized round fibers. It is compared to exact results. Finally, brief comments are made in Section VI concerning the applicability of quasi-ray optics techniques in evaluating the coupling between irregular oversized fibers and the effect of bending. A few general results that do not seem available in convenient form in the literature are derived in the appendices.

II. FAST COUPLING

Solution of the coupled-mode equations when the axial wave numbers of the isolated guides are constant, or vary linearly with z, is recalled in Appendix A. In the present section, only the results are given.

Let us first assume that the coupling c between the two guides and the axial wave numbers k_1, k_2, of the isolated guides is constant (independent of z). Let a power unity be fed into guide 1 at $z = 0$ and the other guide, guide 2, be unexcited. The power in guide 2 grows, at first, according to the law (see Appendix A)

$$P_2(z) = (cz)^2. \tag{1}$$

This result is valid only as long as $\Delta z \ll 1$, where we have defined

$$\Delta \equiv \{[(k_1 - k_2)^2/4] + c^2\}^{\frac{1}{2}}. \tag{2}$$

For example, a -20-dB crosstalk ($P_2 = 0.01$) over a 1-km length of cable is obtained, according to (1), if $c = 10^{-4}$ m^{-1}. Condition $\Delta z \ll 1$ is, for identical guides, $z \ll 10$ km. However, if $(k_1 - k_2)/(k_1 + k_2) = 10^{-4}$, law (1) is applicable only if $z \ll 1$ mm, a drastically different condition. In Section III, the distance between the guides that corresponds to this particular coupling is evaluated.

Now let $k_1 - k_2$ vary linearly with z. The coupling c remains a constant. We write

$$k_1(z) = k_0 + \alpha z, \qquad k_2(z) = k_0 - \alpha z, \tag{3}$$

where k_0 and α denote constants. At large $|z|$, the coupling is insignificant because of the large value of $k_1 - k_2$. The coupling becomes important only near the origin, $z = 0$, where near-synchronism is achieved. Let a power unity be fed into guide 1, at large negative z. The power transferred to guide 2 at large positive z is exactly (see Appendix A)

$$P_2 = 1 - \exp(-\pi c^2/\alpha). \tag{4}$$

We are interested in the case where the k's are crossing very rapidly. Thus, let us assume that α is large and that, consequently, $\pi c^2/\alpha$ is small. In that approximation,

$$P_2 = \pi c^2/\alpha \ll 1. \tag{5}$$

In most practical systems, $k_1 - k_2$ oscillates as a function of z. A significant amount of coupling between two guides takes place only near the crossing points. To develop an understanding of the effects of longitudinal variations of the difference of the axial wave numbers $k_1(z)$ and $k_2(z)$, we model the difference in wave numbers as a simple sinusoid, i.e.,

$$\tfrac{1}{2}(k_1 - k_2) = \delta \sin(\Omega z), \tag{6}$$

where δ denotes the peak deviation of $(k_1 - k_2)/2$ and $2\pi/\Omega$ the period of oscillation. It seems reasonable to assume that the phases of the signals picked up by fiber 2 at the successive crossing points are un-

correlated and that, consequently, the powers add up. This incoherency is a consequence of the fluctuations of the phase of the optical field between successive crossing points. According to (6), the slope α introduced in (3) is

$$\alpha = \delta\Omega. \tag{7}$$

The number of crossing points over a length z is $\Omega z/\pi$. Thus, the power collected by guide 2 over length z is

$$P_2 = (\Omega z/\pi)(\pi c^2)/(\delta\Omega) = c^2 z/\delta. \tag{8}$$

Note that P_2 is independent of Ω. P_2 is proportional to c^2, as was the case in the absence of fluctuations, but it varies linearly with z rather than being proportional to z^2. Let us compare P_2 in (8) and P_2 in (1). The ratio of these two collected powers is

$$\frac{P_2 \text{ (uniform fibers)}}{P_2 \text{ (nonuniform fibers)}} = \delta z. \tag{9}$$

It seems reasonable to assume that, over a length of 1 km ($z = 10^9$ μm), the relative variations of the axial wave number are larger than 10^{-4}: $\delta/k > 10^{-4}$. For the single-mode slab considered in the next section, this number corresponds to a fluctuation of the slab thickness of 0.01 μm. Because k is of the order of 2π μm^{-1}, the reduction in coupling owing to the lack of identity between the two slabs is, in that case, of the order of 50 dB. The results obtained are therefore much too conservative if we assume that the optical guides are identical in evaluating the crosstalk.

III. EVALUATION OF COUPLING BETWEEN TWO SLABS

Let us consider two identical dielectric slabs having thickness $2d$ and material free wave number k. The free wave number in the medium between the slabs (cladding) is denoted k_s, and the spacing between the slabs is denoted $2D$. (See Fig. 1. The intermediate layer is to be ignored for the moment.) The expression for the coupling c between the fundamental H waves is well known (see, for example, Ref. 1):

$$c = \kappa R \exp(-2\kappa D), \tag{10}$$

where

$$\kappa \equiv (k_z^2 - k_s^2)^{\frac{1}{2}} \tag{11a}$$

$$R = (k_z d)^{-1}[1 + (1/\kappa d)]^{-1}[1 - (\kappa^2 d^2/F^2)] \tag{11b}$$

$$F^2 \equiv (k^2 - k_s^2)d^2. \tag{11c}$$

k_z denotes the axial wave number of the isolated slabs (previously denoted k_1 and k_2 for the two waveguides). If we require that only one H mode propagate (for simplicity, we shall ignore the E waves), the

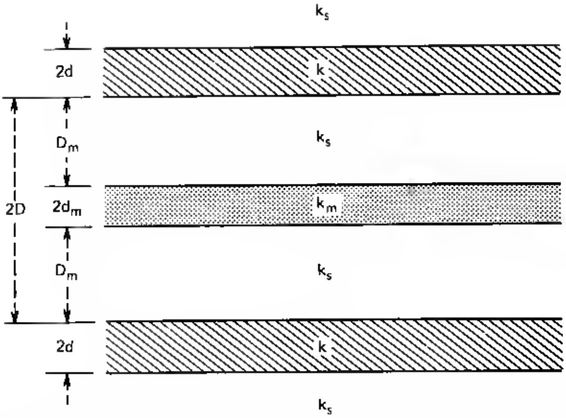

Fig. 1—Coupled dielectric slabs with thickness $2d$ and free wave number k. The cladding medium has free wave number k_s. Crosstalk can be reduced by introducing a metallic layer with free wave number k_m (almost purely imaginary) and thickness $2d_m$.

maximum value of F is $\pi/2$. The theory of dielectric slabs shows that, for that value of F, $\kappa d = 1.28$. Thus, the coupling is

$$c = (0.24/kd^2) \exp(-2.56D/d), \tag{12}$$

where we have made the approximation $k_z \approx k$ in the first term. Thus, for a constant relative spacing D/d, the coupling between two single-mode slabs varies as the inverse of the square of their thickness.

Let us evaluate c for the numerical values

$$2d = 1.32\ \mu\text{m}, \qquad k = 2\pi \times 1.45\ \mu\text{m}^{-1}, \qquad k_s = 2\pi \times 1.4\ \mu\text{m}^{-1}. \tag{13}$$

Thus,

$$F = \pi/2, \qquad k_z d \approx kd = 8.88. \tag{14}$$

If we substitute these results in (11b) and (12), we obtain

$$R = 0.021, \quad c(\text{in m}^{-1}) = 4 \times 10^4 \exp(-3.88D), \tag{15}$$

where D is in μm. If the slabs are identical, -20-dB crosstalk in 1 km is obtained, as we have seen in Section II, when $c = 10^{-4}\ \text{m}^{-1}$. This corresponds, according to (15), to a spacing

$$2D = 11\ \mu\text{m}. \tag{16}$$

If the slabs have some irregularities, with $\delta/k = 10^{-4}$ (corresponding to a variation of slab thickness of 0.01 μm), -20-dB crosstalk is obtained when $c = 0.25\ \text{m}^{-1}$. This coupling corresponds to a smaller spacing: $2D = 6.2\ \mu$m. If the slab thickness is chosen equal to 10.5 μm, keeping $F = \pi/2$ ($\Delta n/n = 5 \times 10^{-4}$), the spacing required for

identical slabs and -20-dB crosstalk over a 1-km length is $2D = 66$ μm, a rather large spacing.

If the two slabs are uniform but are made deliberately dissimilar, a lower crosstalk is obtained. The relative difference δ/k in axial wave numbers is approximately $0.5\ (\Delta d/d)/(kd)^2$, where $\Delta d/d$ is the relative difference in thickness of the two slabs ($F = \pi/2$). For example, if one slab has a thickness $2d$ equal to 1.32 μm and the other has a thickness equal to 1.2 μm, the relative difference in k_z is: $\delta/k = 0.65 \times 10^{-3}$. The maximum relative power that can be transferred from one slab to the other is, according to eq. (39), equal to $(c/\Delta)^2$, where $\Delta \approx \delta$. Thus, a -20-dB crosstalk corresponds, for the above value of δ, to a coupling $c = \delta/10 = 580\ \text{m}^{-1}$. The slab spacing $2D$ corresponding to that coupling is given by (15). We obtain $2D = 2.2\ \mu$m. More generally, we find that $D \approx 1.5d$ for any d, if F is kept equal to $\pi/2$ and $\Delta d/d = 0.1$. Thus, a considerable reduction in spacing is tolerable, in principle, if the slabs are made dissimilar. However, fast fluctuations along the z-axis of the slab dimensions with a period of the order of $\pi/\delta \approx 100\ \mu$m would reestablish synchronism between the two slabs. Fluctuations that are too small in amplitude to deteriorate the propagation under normal conditions (e.g., no significant coupling to the radiation modes) may nevertheless introduce a large crosstalk when the slabs are very close to each other. Thus, the result obtained above, that the spacing between slabs can be reduced to $1.5 \times (2d)$ if the fibers are made dissimilar, may not hold true in practical conditions.

IV. TRANSMISSION THROUGH A METALLIC LAYER UNDER EVANESCENT WAVE EXCITATION

The results in Section II show that the crosstalk power P_2 is proportional to the square of the coupling c. We have shown in Ref. 1 that, for identical slabs and a symmetrical configuration, the coupling c is proportional to the square of the normalized field halfway between the two slabs. Thus, the crosstalk is proportional to the fourth power of the normalized field halfway between the two slabs. If we introduce a metallic layer of thickness $2d_m$, symmetrically centered between the two slabs as shown in Fig. 1, the crosstalk is reduced in proportion to the fourth power of the field in the middle of the metallic layer. This field reduction, denoted t (for transmission), is evaluated in the present section.

Let us consider an evanescent wave with axial wave number $k_z > k_s$, where k_s denotes the free wave number in the medium. This wave decays in the x direction according to

$$E(x) = E_0 \exp(-\kappa x), \tag{17}$$

$$\kappa \approx (k_z^2 - k_s^2)^{\frac{1}{2}}. \tag{18}$$

Let us now introduce a metallic layer with complex wave number $k_m \equiv k_{mr} + ik_{mi}$ and thickness $2d_m$. The ratio t of the field in the middle of the layer to the field at the same point in the absence of the layer is derived in Appendix B. Provided the layer is sufficiently thick or, more precisely, that

$$\text{Real } (\kappa_m d_m) \gg 1, \tag{19}$$

where

$$\kappa_m \equiv (k_z^2 - k_m^2)^{\frac{1}{2}}, \tag{20}$$

we have

$$t = [4\kappa\kappa_m/(\kappa + \kappa_m)^2] \exp [(\kappa - \kappa_m)d_m]. \tag{21}$$

At a free-space wavelength $\lambda_0 = 1\ \mu\text{m}$, $k_0 = 2\pi\ \mu\text{m}^{-1}$, the wave number of silver is almost purely imaginary,[3]

$$k_m^2 \equiv (k_{mr} + ik_{mi})^2 = (0.2k_0 + i5k_0)^2 = -985 + 79i \text{ (in } \mu\text{m}^{-2}), \tag{22}$$

and, for a typical glass, assumed lossless ($n_s = 1.4$),

$$k_s^2 = n_s^2 k_0^2 = (1.4k_0)^2 = 77.4\ \mu\text{m}^{-2}. \tag{23}$$

With the value of $\kappa^2 \equiv k_z^2 - k_s^2 = 3.76\ \mu\text{m}^{-2}$ in (14), and k_m^2, k_s^2 in (22) and (23), we obtain $\kappa_m = 32 - 1.3i$, and, from (21), a power transmission

$$T \equiv tt^* = 0.062 \exp (-60d_m), \tag{24}$$

where d_m is in μm, provided

$$d_m \gg 0.03\ \mu\text{m}. \tag{25}$$

Because the crosstalk power P_2 is proportional to the square of the power transmission T, the introduction of a layer of silver of thickness $2d_m$ between the two slabs reduces the crosstalk in dB by

$$20 \log_{10} (T) = 520d_m, \tag{26}$$

where d_m is in μm. For example, if the layer thickness is $2d_m = 0.5\ \mu$m, the crosstalk is reduced by 130 dB. This reduction is independent of the initial value of the crosstalk, within the approximations made. Thus, a 0.5-μm-thick layer of silver is sufficient to ensure a complete isolation of adjacent fibers, at a wavelength $\lambda_0 = 1\ \mu$m.

Surface polaritons can be guided near the dielectric ($k_s^2 > 0$) and metallic ($k_m^2 < 0$) interface. However, the losses of such modes are extremely high over a distance of 1 km. The cladding modes are also strongly attenuated, and it seems that they can be safely ignored. For comparison, let us consider, in place of the metallic layer, a low-index plastic material of the Teflon type, with a refractive index $n = 1.32$. We now have $k_m^2 = 69\ \mu\text{m}^{-2}$ and $\kappa_m = 3.47\ \mu\text{m}^{-1}$. We obtain a cross-

talk reduction equal to $26d_m$ in dB, where d_m is in μm. Thus, a 50-dB reduction in crosstalk requires a 4-μm-thick layer of low-index plastic material.

V. LOSS INTRODUCED BY A METALLIC LAYER

We are now concerned with the fact that, because the refractive index of a metal is not purely imaginary, the presence of the metallic layer may increase significantly the loss of the modes guided by the fibers. This loss depends critically on the distance between the metallic layer and the fibers and, therefore, on the distance between the two fibers. The loss suffered by the fiber is influenced by the complex reflection of the metallic layer for evanescent waves. This reflection, strictly speaking, depends on the thickness of the metallic layer. Exact expressions are given in Appendix B. However, in all our numerical examples, the thickness of the metallic layer is so large that it can be assumed infinite. In that case, the reflection r reduces to

$$r = (\kappa - \kappa_m)/(\kappa + \kappa_m), \tag{27}$$

where κ and κ_m are defined in (18) and (20), respectively. Because the imaginary part κ_{mi} of κ_m is much smaller than the real part κ_{mr}, the imaginary part r_i of r is approximately

$$r_i \approx 2\kappa\kappa_{mi}/\kappa_{mr}^2. \tag{28}$$

If we use for k_z, k_s, and k_m the numerical values in (14), (23), and (22), respectively, we find $r_i = 0.005$.

To obtain the loss suffered by the slab, we use the perturbation formula derived in Appendix C. The variation of k_z is assumed to be small. The variation of the field near the perturbing object, however, is not assumed small. In the present case, k_z is real before perturbation. The introduction of the metallic layer causes k_z to acquire a small imaginary part, k_{zi}. The imaginary part k_{zi} of k_z is the fiber loss, in neper/unit length. There is also a small variation of the real part of k_z. This variation, however, is of no interest to us. We have (see Appendix C)

$$k_{zi} = r_i\kappa R \exp(-2\kappa D_m), \tag{29}$$

where R is the slab parameter defined in (12a) and D_m the distance between the slab and the metallic layer. The imaginary part r_i of the metallic layer reflectivity is given in (28).

For the numerical values used earlier in (14) and (15), we obtain from (29)

$$\mathcal{L}_{dB/km} = 8.7 \times 10^3 k_{zi} = 2.6 \times 10^6 \times \exp(-3.88 D_m), \tag{30}$$

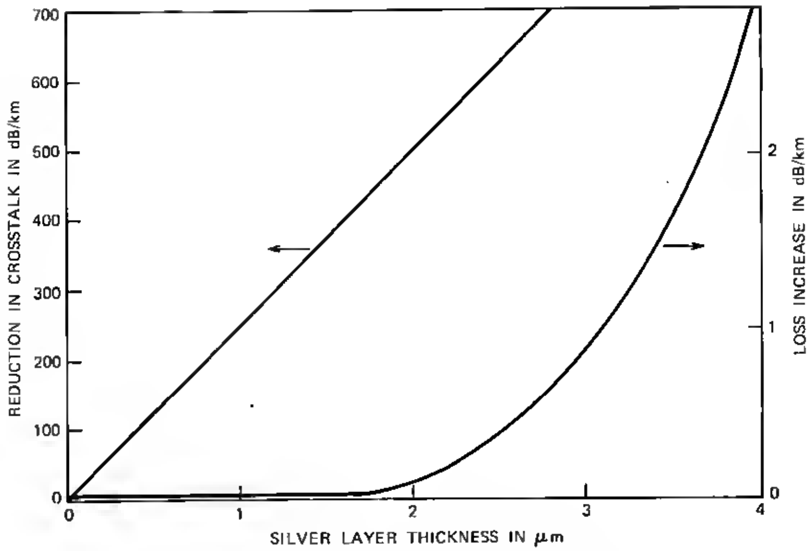

Fig. 2—Reduction in crosstalk and increase in fiber loss resulting from the introduction of a silver layer of thickness $2d_m$ (free-space wavelength = 1 μm). The dielectric slabs have a normalized frequency $F \equiv (k^2 - k_s^2)^{\frac{1}{2}} d = \pi/2$. Their spacing is kept equal to 11 μm. The loss varies with d_m only because of the change in the slab-layer spacing. In the absence of metallic layer, crosstalk is −20 dB/km.

where D_m is in μm. For $D_m = D - d_m = 5.25$ μm, the loss introduced by the metallic layer, given in (30), is only

$$\mathcal{L} = 0.017 \text{ dB/km}. \tag{31}$$

This loss is quite negligible compared with the other losses (absorption because of impurity or scattering losses) suffered by the wave. However, because $\mathcal{L}$ depends critically on D_m, this loss may not be negligible in all practical cases. The reduction of the crosstalk and the increase of loss caused by a silver layer of thickness $2d_m$ are shown in Fig. 2 for the dielectric slabs considered earlier, as functions of $2d_m$. Note that, if we assume for simplicity that the thickness of the metallic layer is negligible compared with the slab spacing ($2d_m \ll 2D$), the (dimensionless) ratio of k_{zi} (loss) and c is, approximately,

$$k_{zi}/c = 2(k_{mr}/k_{mi}^2)\kappa^2 d. \tag{32}$$

Thus, the best metal, from the point of view of propagation, is the one whose k_{mr}/k_{mi}^2 is the smallest.

VI. ROUND FIBERS

The general coupling formula in Ref. 1 is applicable, in principle, to round fibers. Round fibers are more often encountered in practice

than are slabs. The geometry is shown in Fig. 3. The fibers are assumed identical, with radius a and spacing $2D$. The results are given only for the scalar fundamental field $\psi \approx \mathrm{HE}_{11}$ of oversized fibers $[F \equiv (k^2 - k_s^2)^{\frac{1}{2}}a \gg 1]$. In that approximation, the normalized field is easily found to be (see Part II of Ref. 1)

$$\psi(y) = u_0(\pi^{\frac{1}{2}}k^{\frac{1}{2}}aF)^{-1} \exp(-Fy^2/2a^2), \tag{33}$$

where $u_0 \approx 2.4 \cdots$ is the first zero of the Bessel function of order zero. The y axis is tangent to the rod considered, as shown in Fig. 3. The Fourier transform of $\psi(y)$ is

$$\begin{aligned}\hat{\psi}(k_y) &= (2\pi)^{-\frac{1}{2}}\int_{-\infty}^{+\infty} \psi(y) \exp(-ik_y)dy \\ &= \pi^{-\frac{1}{2}}u_0k^{-\frac{1}{2}}F^{-\frac{3}{2}} \exp(-k_y^2a^2/2F).\end{aligned} \tag{34}$$

Because the spectral component $\hat{\psi}(k_y)$ varies approximately as $\exp(-sx)$ as a function of x, where $s \equiv (k_z^2 - k_s^2)^{\frac{1}{2}} \approx F/a$, the

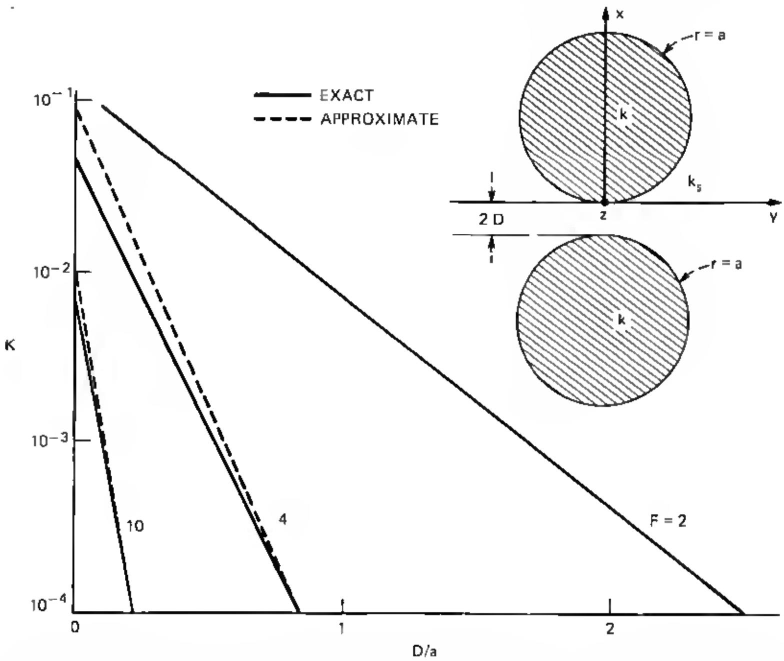

Fig. 3—Variation of the coupling between two dielectric rods of radii a as a function of their spacing ($2D$). The dimensions and free wave numbers are shown. The parameter K is defined as $ca(1 - k_s^2/k^2)^{-\frac{1}{2}}$, and c is the coupling. The plain lines are from Snyder exact theory,[7] and the dashed lines from the theory in Ref. 1, applied to large normalized frequencies F.

coupling is

$$c = \int_{-\infty}^{+\infty} s(k_y)\hat{\psi}_2^*(k_y)\hat{\psi}_1(k_y)dk_y$$
$$= (u_0^2/\pi^{\frac{1}{2}})k^{-1}F^{-\frac{3}{2}}a^{-2}\exp(-2FD/a). \tag{35a}$$

In place of c, we can use a normalized coupling K defined by

$$K = ca[1 - (k_s^2/k^2)]^{-\frac{1}{2}}. \tag{35b}$$

In the general expression for c in (34), $\hat{\psi}_2$ and $\hat{\psi}_1$ represent the spectral components of the field of the two fibers along the y-axis at $x = 0$. The normalized coupling K is plotted in Fig. 3 (dashed lines) as a function of the ratio D/a of the fiber spacing ($2D$) to fiber diameter ($2a$). In that figure, the parameter is the normalized frequency F. For comparison, an exact result obtained by Snyder[4] is shown as a plain line. The agreement is very good for $F \gtrsim 4$.

The advantage of the method used in this section is that it is applicable when the two fibers are separated by a metallic layer. In that case, one need only introduce inside the integral sign in the first expression in (34) a term $T(k_y)$, where T denotes the power transmission of the metallic layer, defined in (24). T now depends sligbtly on k_y because, in the expressions given earlier for T, the axial wave number k_z should be replaced by $(k_z^2 + k_y^2)^{\frac{1}{2}}$. The effect of the dependence of T on k_y is small, however, and the value obtained earlier for T for slabs is approximately applicable to round fibers as well.

VII. MULTIMODED IRREGULAR FIBERS

We shall make only qualitative comments. In the preceding calculations, we have considered the coupling between one mode of one core and one mode of another adjacent core. If the cores can carry many modes and have dimensions that fluctuate as a function of z, with such an amplitude that the variations in axial wave numbers exceed the spacing (in axial wave numbers) between adjacent modes, some averaging takes place. The situation becomes comparable, at least over some distance, to that of a slab radiating power into a semi-infinite dielectric, a situation discussed in detail in Part II of this series of papers.[1]

Let us picture the field in slab 1 (excited at $z = 0$) as made up of two plane waves. The plane wave moving toward slab 2 tunnels into slab 2. Because of the fluctuations in axial wave numbers, the power transferred from slab 1 to slab 2 is essentially the power carried by that tunnelling wave; we can ignore the fact that this wave, after tunnelling, is reflected back and forth inside slab 2 and may tunnel back to slab 1. The power transferred from slab 1 to slab 2, then, is

proportional to z, for small z, rather than to the square of z, as is the case in the absence of irregularities. This picture is consistent with that used by Cherin,[5] who adds the powers transmitted by tunnelling rays. Let us emphasize that the validity of this quasi-ray optics approach rests on the presence of large slow fluctuations of the core dimensions. A simple calculation shows that the relative fluctuations of the slab thickness must exceed the reciprocal of the mode number. This condition is never met for the low-order modes, but it may be met by the higher-order modes. Thus, the situation is rather complicated and requires a deeper analysis. This quasi-ray technique should not be confused with that of Kapany and Burke,[6] where the slabs are assumed identical and the *fields* of the tunnelling rays, rather than their powers, are added. In the preceding discussion, we have assumed that the fiber cable is essentially straight. The coupling increases significantly if the cable is bent.[7] This effect makes it even more important to provide shields between adjacent fibers.

VIII. CONCLUSION

We have shown that a drastic reduction of crosstalk between parallel dielectric slabs can be obtained by introducing a layer of silver (thickness $\approx 0.5\ \mu$m) between adjacent slabs. The reduction, in decibels, is proportional to the imaginary part of the refractive index of the metallic layer and to the layer thickness. In many cases of practical importance, the loss introduced by this metallic layer is negligible. We have also shown that, because of unavoidable irregularities in the fiber dimensions, the crosstalk is at least 40 dB below that expected for identical fibers.

IX. ACKNOWLEDGMENTS

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APPENDIX A

Fast and Adiabatic Coupling

Let ψ denote the field of a guide, such that $\psi\psi^*$ is the power. When two guides are weakly coupled, their respective fields ψ_1, ψ_2 approximately satisfy the well-known equations[8]

$$\begin{aligned} -id\psi_1/dz &= k_1(z)\psi_1 + c\psi_2 \\ -id\psi_2/dz &= k_2(z)\psi_2 + c\psi_1. \end{aligned} \tag{36}$$

For simplicity, we assume that the axial wave numbers k_1, k_2 of the isolated guides are real and that the coupling c is a real constant. The solution when k_1, k_2 are constant is well known. For the convenience

of the reader, this solution is derived below. The general solution of (36) is a superposition of normal modes

$$\begin{aligned} \psi_1(z) &= \psi_1^+ \exp(ik^+z) + \psi_1^- \exp(ik^-z) \\ \psi_2(z) &= \psi_2^+ \exp(ik^+z) + \psi_2^- \exp(ik^-z), \end{aligned} \tag{37}$$

where

$$k^\pm = (k_1 - k_2)/2 \pm \Delta \tag{38a}$$

$$\Delta = \{[(k_1 - k_2)^2/4] + c^2\}^{\frac{1}{2}}. \tag{38b}$$

If the initial conditions are $\psi_1(0) = 1$, $\psi_2(0) = 0$, that is, if only guide 1 is excited at the origin ($z = 0$), the field in the unexcited guide, 2, is

$$\psi_2(z) = (ic/\Delta) \exp[i(k_1 + k_2)z/2] \sin(\Delta z). \tag{39}$$

Thus, for small z, the power in guide 2 increases as

$$P_2(z) = (cz)^2, \qquad \Delta z \ll 1. \tag{40}$$

This result is independent of $k_1 - k_2$. See Fig. 4.

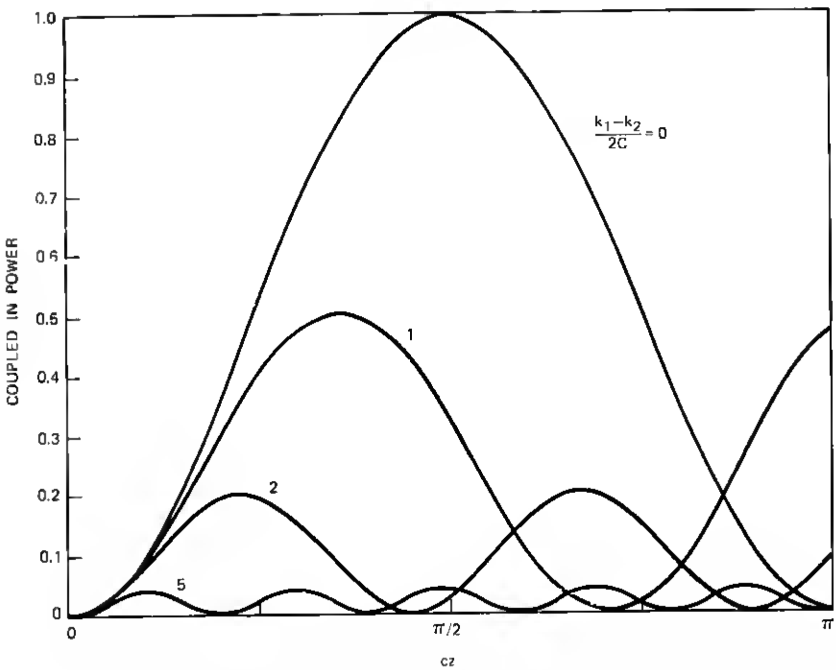

Fig. 4—Variation of the optical power picked up by fiber 2, where only fiber 1 is excited at $z = 0$, as a function of the normalized axial distance. The axial wave numbers of the isolated fibers are assumed to be constant but different [parameter $(k_1 - k_2)/2c$]. Note that the behavior for small cz is independent of $k_1 - k_2$.

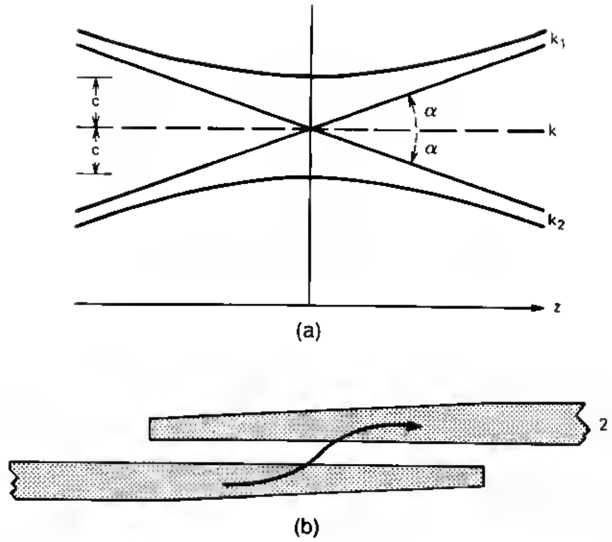

Fig. 5—(a) Linear variation of the axial wave number of the isolated waveguides as a function of the axial coordinate z. The hyperbolas represent the normal mode wave numbers. (b) Adiabatic coupling in fiber optics. All the power from one fiber is transferred to the other fiber if the k's vary sufficiently slowly. This principle is applicable to multimode fibers.

Let now the axial-wave numbers k_1, k_2 of the isolated guides vary linearly with z

$$k_1(z) = k_0 + \alpha z, \qquad k_2(z) = k_0 - \alpha z, \tag{41}$$

where k_0 and α denote constants. Synchronism takes place only near the origin, $z = 0$. Let us set

$$\psi_{1,2}(z) = A_{1,2}(z) \exp{(ik_0 z)} \tag{42}$$

in (36). After differentiation and substitution, we obtain an equation for A_1,

$$(d^2A_1/dz^2) + (\alpha^2 z^2 + c^2 - i\alpha)A_1 = 0. \tag{43a}$$

A similar equation holds for A_2 that we need not write down. Equation (43a) is the equation for parabolic cylinder functions. The asymptotic form of the solution, valid for $-\pi/2 \leqq \arg(z) \leqq \pi$ is, for a power unity at $z = -\infty$ (see Ref. 9),

$$A_1(z) = \exp{[i(\alpha/2)z^2 + i(c^2/2\alpha) \log{(-z)}]}, \qquad z \ll c/\alpha \tag{43b}$$

$$A_1(z) = \exp{[i(\alpha/2)z^2 + i(c^2/2\alpha) \log{(z)} - \pi c^2/2\alpha]}, \qquad z \gg c/\alpha, \tag{43c}$$

as we easily verify by substituting (43b) in (43c) and neglecting terms of order z^{-2}. To go from (43b) to (43c), note that $\log{(-z)}$

$= i\pi + \log(z)$. Note also that a change in the unit with which z is measured affects only the amplitude of A_1, which is arbitrary.

The power in guide 2 after the interaction has taken place, that is, for large positive z, is, according to (43c),

$$P_2 = 1 - A_1 A_1^* = 1 - \exp(-\pi c^2/\alpha). \tag{44}$$

Let us first assume that $\pi c^2/\alpha$ is very small compared witb unity, that is, the k's are crossing very rapidly. In that case, guide 1 transfers only a small amount of power to guide 2, equal to $\pi c^2/\alpha$. This is the result used in the text.

When $\pi c^2/\alpha$ is very large compared with unity, that is, when the variation of $k_1 - k_2$ is very slow, almost all the power from guide 1 is coupled to guide 2. This is the principle of the Cook adiabatic coupler.[10] This mechanism is applicable also to multimode dielectric waveguides. It may be used to couple two optical fibers because the dimensions are not critical. Only slowness is required.[11] (See Fig. 5.)

APPENDIX B

Transmission and Reflection at a Metallic Layer Under Evanescent Wava Excitation

Let the metallic layer have a complex free wave number $k_m \equiv k_{mr} + ik_{mi}$ and a thickness d_m. The surrounding medium is assumed to have a real free wave number k_s. The field has the general form (see Fig. 6)

$$E(x) = \begin{cases} E_0[\exp(-\kappa x) + r\exp(\kappa x)], & x \leqq 0 \\ E^- \exp(-\kappa_m x) + E^+ \exp(\kappa_m x) & 0 \leqq x \leqq d_m \\ E_0 t \exp(-\kappa x) & x \geqq d_m, \end{cases} \tag{45}$$

where

$$\kappa \equiv (k_z^2 - k_s^2)^{\frac{1}{2}} \tag{46}$$

is real, and

$$\kappa_m \equiv (k_z^2 - k_m^2)^{\frac{1}{2}}. \tag{47}$$

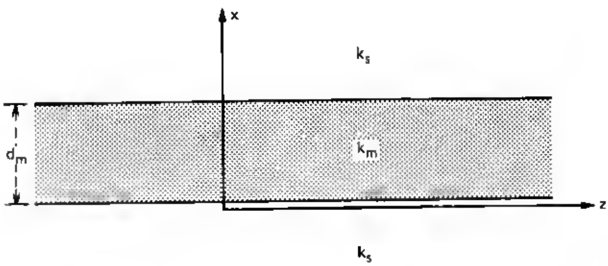

Fig. 6—Transmission of a metallic layer with thickness d_m and free wave number k_m under evanescent wave excitation (axial wave number $k_z > k_s$). At large negative x, the field is assumed unperturbed by the layer.

The axial wave number k_z is assumed to he real and larger than k_s. By specifying that E and dE/dx are continuous at the boundaries, $x = 0$, $x = d_m$, we ohtain the reflection r and the transmission t:

$$r = [(\kappa/\kappa_m) - (\kappa_m/\kappa)][2 \coth(\kappa_m d_m) + (\kappa_m/\kappa) + (\kappa/\kappa_m)]^{-1} \tag{48}$$

$$t = \exp(\kappa d_m)\{\cosh(\kappa_m d_m) + \tfrac{1}{2}[(\kappa_m/\kappa) + (\kappa/\kappa_m)] \sinh(\kappa_m d_m)\}^{-1}. \tag{49}$$

We shall now assume that the metallic layer is thick in the sense that Real $(\kappa_m d_m) \gg 1$. These conditions are well satisfied for the metallic layers that we consider in the main text. In that case, (48) and (49) reduce to

$$r = (\kappa - \kappa_m)/(\kappa + \kappa_m) \tag{50}$$

$$t = [4\kappa\kappa_m/(\kappa + \kappa_m)^2] \exp[(\kappa - \kappa_m)d_m], \tag{51}$$

respectively. Equations (50) and (51) are the results used in the text.

APPENDIX C

Loss Introduced by a Metallic Layer

Let us consider a uniform reciprocal waveguide and let a uniform rod he introduced that perturbs the propagation of the waveguide (Fig. 7a). We assume that the perturbing rod does not support trapped modes or, if it does, that the axial wave numhers of these trapped modes are sufficiently far away from that, k_{z0}, of the waveguide. No resonant coupling is assumed to take place.

We shall first recall a very general result. Let $\mathbf{E}^+$, $\mathbf{H}^+$ and $\mathbf{E}_p$, $\mathbf{H}_p$ denote two time-harmonic fields at the same frequency in the same medium. If we assume that the medium is reciprocal (that is, that the tensor permittivity is symmetrical), it readily follows from the Maxwell equations that the divergence of the vector

$$\mathbf{J} = \mathbf{E}^+ \times \mathbf{H}_p + \mathbf{H}^+ \times \mathbf{E}_p \tag{52}$$

is equal to zero. Thus, the flux of $\mathbf{J}$ through any closed surface is equal to zero. In what follows, an $\exp(-i\omega t)$ term is omitted.

Now let $\mathbf{E}^+$, $\mathbf{H}^+$ he the field propagating in the $-z$ direction along an open waveguide. The dependence of $\mathbf{E}^+$ and $\mathbf{H}^+$ on z is denoted: $\exp(-ik_{z0}z)$. Let $\mathbf{E}_p$, $\mathbf{H}_p$ he the field propagating in the $+z$ direction in the presence of the perturhing rod with an $\exp(ik_z z)$ dependence on z. The closed surface S is taken as the surface shown in Fig. 7a bounded by the planes $z = 0$ and $z = dz$, the volume of the perturbing rod being excluded. For that choice, the medium enclosed by S is the same for both fields. We can therefore use the result stated earlier that the flux of $\mathbf{J}$ through S is zero. Let us consider the various contributions to that flux. The flux of $\mathbf{J}$ through the plane $z = dz$ differs from the

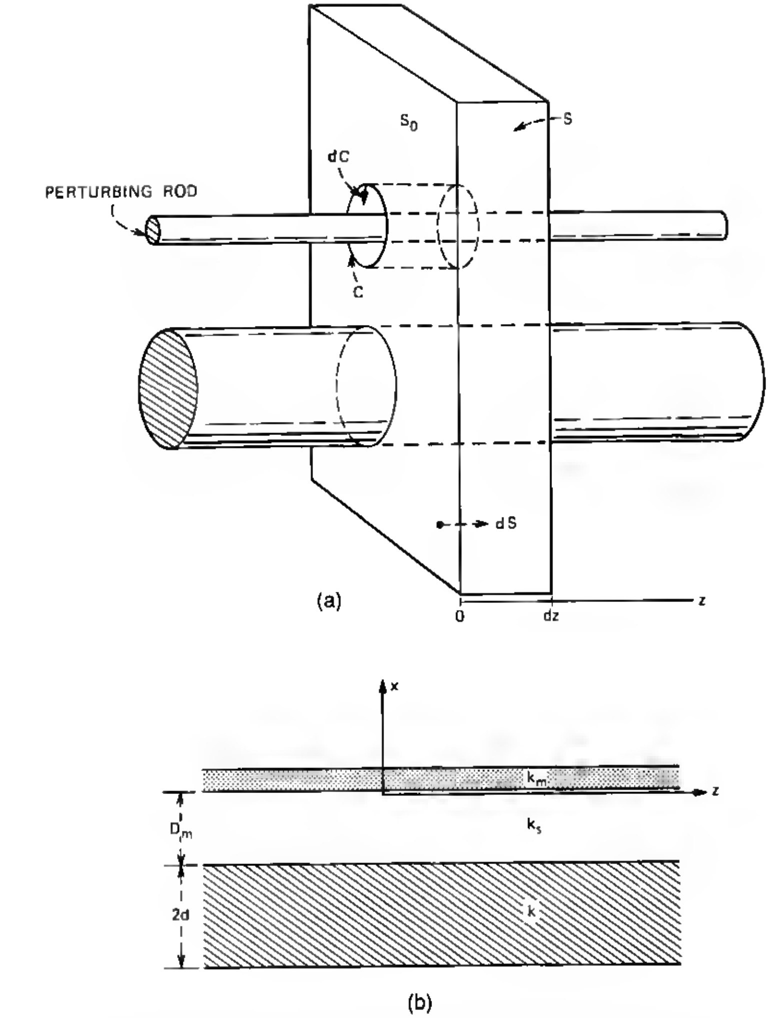

Fig. 7—(a) Schematic for the derivation of the general perturbation formula. The dielectric waveguide is perturbed by a small lossy rod. Tbe closed surface S extends to infinity wbere the fields considered are assumed to vanish. (b) Application to the perturbation of H waves guided by a dielectric slab (k) by a lossy slab (k_m).

flux of $\mathbf{J}$ through the plane $z = 0$ only by a factor $-\exp[i(k_z - k_{z0})dz]$. The difference between these fluxes is, therefore, $i(k_z - k_{z0})dz$ for small dz. Because we are considering only trapped modes, the flux at infinity is zero. The flux through the surface surrounding the perturbing rod is dz times the line integral of $\mathbf{J}\cdot\mathbf{d}C$, with $\mathbf{d}C$ a vector perpendicular to the contour surrounding the rod, pointing inward, whose

length is the elementary arc length. Thus, we bave exactly

$$i\Delta k_z = \int_C \mathbf{J}\cdot d\mathbf{C} \Big/ \int_{S_0} \mathbf{J}\cdot d\mathbf{S}, \tag{53}$$

where

$$\Delta k_z \equiv k_z - k_{z0}. \tag{54}$$

S_0 denotes the transverse plane, $z = 0$ minus the area enclosed by C, and $d\mathbf{S}$ denotes a vector directed along tbe z axis whose length is the elementary area. The derivation given above is almost identical to that in Ref. 1 for coupled waveguides. We now assume that the perturbation is small. Thus, we can replace $\mathbf{E}_p$, $\mathbf{H}_p$ by the unperturbed field $\mathbf{E}$, $\mathbf{H}$ propagating in tbe $+z$ direction in the integral over S_0 in (53). Tbis is not permissible, however, for the integral over C, in general.

Let (52) be specialized to the H waves guided by a dielectric slab shown in Fig. 7b. In that case, $\mathbf{E}$ has only one component: $E_y \equiv E(x)$, $H_z = (1/i\omega\mu_0)\partial E/\partial x$, and $H_x = -(k_z/\omega\mu_0)E$. Taking into account $E_y^+ = E_y$ and $H_z^+ = H_z$ (see Ref. 1), we obtain

$$\Delta k_z = [(E\partial E_p/\partial x) - (E_p\partial E/\partial x)] \Big/ \left(2k_z \int_{-\infty}^{+\infty} E^2 dx\right), \tag{55}$$

where we bave assumed that E_p differs significantly from E only near the perturbing slab. The unperturbed field is, for $-D_m < x < 0$,

$$E = \exp(-\kappa x), \tag{56}$$

and tbe perturbed field is tbat given in (45)

$$E_p = \exp(-\kappa x) + r \exp(\kappa x), \tag{57}$$

where

$$\kappa \equiv (k_z^2 - k_3^2)^{\frac{1}{2}}.$$

The amplitudes in (56) and (57) are so chosen that $E_p \approx E$ for large negative x, e.g., $x = -D_m$.

We first evaluate

$$(E\partial E_p/\partial x) - (E_p\partial E/\partial x) = 2r\kappa, \tag{58}$$

where we have used (56) and (57). Note that the result (58) is independent of x (for $-D_m < x < 0$). Substituting (58) in (55), the imaginary part of k_z is found

$$k_{zi} = r_i\kappa R \exp(-2\kappa D_m), \tag{59}$$

where r_i denotes the imaginary part of r, evaluated in Appendix B. We have introduced in (59) the field strength parameter

$$R = \left(k_z \int_{-\infty}^{+\infty} E^2 dx\right)^{-1}. \tag{60}$$

In the above definition of R, the field is assumed to be unity at the guide-cladding boundary. For a dielectric slab, the value of R is given in (12). Equation (59) is the result used in the main text.

REFERENCES

1. J. A. Arnaud, "Transverse Coupling in Fiber Optics—Part I: Coupling Between Trapped Modes," B.S.T.J., *53*, No. 2 (February 1974), pp. 217–224; "Transverse Coupling in Fiber Optics—Part II: Coupling to Mode Sinks," B.S.T.J., *53*, No. 4 (April 1974), pp. 675–696.
2. D. Marcuse, "The Coupling of Degenerate Modes in Two Parallel Dielectric Waveguides," B.S.T.J., *50*, No. 6 (July–August 1971), pp. 1791–1816.
3. M. Born and E. Wolf, *Principles of Optics*, New York: Pergamon Press, 1965, p. 623.
4. A. W. Snyder, "Leaky-Ray Theory of Optical Waveguides of Circular Cross Section," Appl. Phys., *4*, No. 4 (September 1974), pp. 273–298.
5. A. H. Cherin and E. J. Murphy, "Quasi-Ray Analysis of Crosstalk Multimode Optical Fibers," B.S.T.J., *54*, No. 1 (January 1975), pp. 17–45.
6. N. S. Kapany and J. J. Burke, *Optical Waveguides*, New York: Academic Press, 1972.
7. E. A. J. Marcatili, "Bends in Optical Dielectric Guides," B.S.T.J., *48*, No. 5 (September 1960), pp. 2103–2132.
8. S. E. Miller, "Coupled Wave Theory and Waveguide Applications," B.S.T.J., *33*, No. 4 (July 1955), pp. 807–822.
9. G. H. Wannier, "Probability of Violation of the Ehrenfest Principle in Fast Passage," Physics, *1*, No. 5 (May-June 1965), pp. 251–253.
10. J. S. Cook, "Tapered Velocity Couplers," B.S.T.J., *34*, No. 4 (July 1955), pp. 807–822.
11. M. G. F. Wilson and G. A. Teh, "Tapered Optical Directional Coupler," IEEE J. of Microwave Theory and Technique, *MTT-23*, No. 1 (January 1975), pp. 85–92.